

Influence of Elastic Shear Stiffness on Richtmyer-Meshkov Instability

*JeeYeon N. Plohr (T-1) and
Bradley J. Plohr (T-13)*

In 1953, R. D. Richtmyer wrote a Los Alamos Scientific Laboratory report, published as Ref. [1], about what happens when a shock wave strikes an interface between two fluids head on. By analogy with the classical Rayleigh-Taylor instability [2, 3] of an interface subjected to constant acceleration, he expected that the impulsive acceleration of the shock wave would cause deviations of the interface from perfect flatness to grow in time. The combined mathematical and computational analysis in Ref. [1] confirmed this expectation. This fluid instability was seen experimentally by E. Meshkov [4] and is now called the Richtmyer-Meshkov instability.

What happens if the two materials are solids rather than fluids? One difference between a solid and a fluid is elastic shear stiffness, which should resist the growth of perturbations. This tendency is seen in the influence of elastic shear stiffness on the Rayleigh-Taylor instability in the small-amplitude regime [5]: the growth rate of perturbations is reduced, and short wavelength perturbations are stabilized altogether. We have investigated the elastic Richtmyer-Meshkov instability in the small amplitude regime. Our finding is that shear stiffness, however small, causes perturbations of any wavelength to remain small: Richtmyer-Meshkov flow for elastic solids is stable.

Richtmyer-Meshkov flow is described by the Euler equations, a set of partial differential equations (PDEs), involving several flow variables (mass density, particle velocity components, and specific energy), that embodies conservation of mass, momentum,

and energy. To determine whether a particular flow (the “background flow”) is unstable, one rewrites the Euler equations in terms of the deviations of the flow variables from their background values (the “perturbation”). So long as the perturbation remains small, the rewritten equations are close to the “linearized equations” that neglect terms of quadratic and higher order in the perturbation. The linearized equations are then studied to see whether an initially small, but nonzero, perturbation grows in time.

For classical instability problems, such as the Rayleigh-Taylor problem, the linearized equations are amenable to exact mathematical analysis, primarily because the coefficients appearing in the linearized equations are independent of space and time. For the Richtmyer-Meshkov problem, however, this is not true; analysis reduces the problem to a PDE for one space and one time dimension, but this PDE has no (known) analytical solutions. Richtmyer’s essential innovation was to use an early digital computer to simulate the reduced PDE. He found that Richtmyer-Meshkov flow for fluids is unstable, with the perturbation amplitude growing roughly linearly with time.

Our work on elastic solids [6, 7] follows Richtmyer’s approach and later refinements [8] but is more complicated because representation of solid deformation requires a nine-component tensor, whereas a single scalar, the volume, suffices for a fluid. A nonstandard formulation of elasticity [9, 10] casts the governing equations as a system of conservation laws. Equations of state for the solids are needed to complete the governing equations; the ones we developed are thermodynamically consistent and allow for arbitrarily large volumetric deformation combined with a small shear deformation. The background flow is constructed by solving the one-dimensional problem of shock transmission/reflection at a material interface, and the initial values for the perturbation are constructed by solving a two-dimensional version of this problem when the angle of incidence is small. The conservation laws are linearized around the background flow, as are the jump conditions, which are

implied by the conservation laws, for the discontinuous waves (the longitudinal and shear shock waves and the material interface). For each sinusoidal mode, we obtain linear conservation laws, with source terms and spatially varying coefficients, in one space and one time dimension. The initial-value problem is solved numerically using a finite difference method supplemented by a front tracking scheme.

The plot of amplitude vs time from our simulations is shown in Fig. 1. Our simulations support the following conclusions concerning the growth rate and amplitude of perturbations of a frictionless material interface between elastic solids when it is struck normally by a shock wave:

- Even a small shear modulus changes the late-time asymptotic behavior of the growth rate. Rather than approaching a constant so that the amplitude grows linearly, it oscillates in such a way that the amplitude remains bounded. In particular, the linear theory remains valid at late time.
- The amplitude oscillates around an asymptotic value with a frequency that grows with the shear moduli (see Fig. 2) and is independent of the strength of the incident shock wave.
- If the shock strength is increased, the amplitude oscillates about a smaller asymptotic value and the oscillations increase in variation.
- Varying the bulk modulus has little effect on the behavior of the material interface.

We explain the striking difference between fluid and elastic solid behavior in Richtmyer-Meshkov flow in the following way. The role of the shock wave in Richtmyer-Meshkov is to deposit vorticity on the perturbed interface. This sheet of vorticity is subject to Kelvin-Helmholtz instability. In (inviscid, incompressible) fluid dynamics, the vorticity remains on the interface, so that Kelvin-Helmholtz instability leads to growth of perturbations. In contrast, for elastic solids, the vorticity propagates at the shear wave speeds and thus escapes the interface, so that perturbations of the interface do not grow.

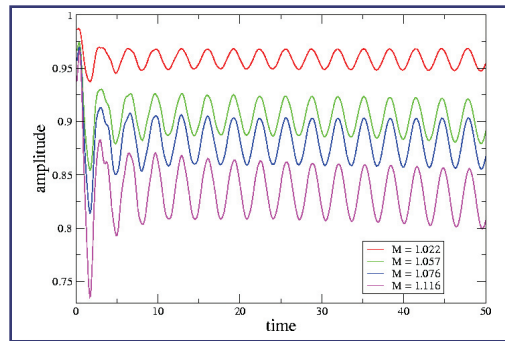


Figure 1—
Amplitude vs time for tantalum/aluminum and various Mach numbers of the incident shock wave.

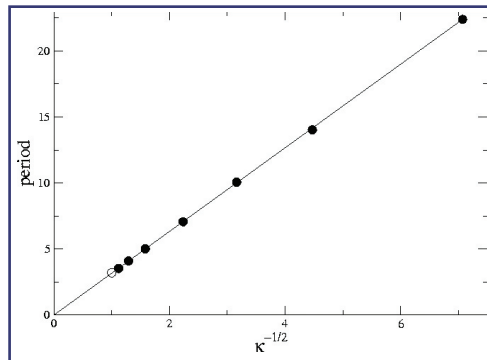


Figure 2—
Period of oscillations vs $1/\sqrt{\kappa}$ for various values of the interpolation parameter κ , which determines the shear moduli κG_{Ta} and κG_{Al} of the model solids.

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For more information, contact JeeYeon Plohr (jplohr@lanl.gov).

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